Let $f(x) = \frac{x}{6-x}$.

SCORE: ______ / 8 PTS

[a] Find $f'(x)$.

$$\boxed{\lim_{h \to 0} \frac{\frac{x+h}{6-(x+h)} - \frac{x}{6-x}}{h}} \cdot \frac{(6-(x+h))(6-x)}{(6-(x+h))(6-x)}$$ 1

$$= \boxed{\lim_{h \to 0} \frac{(x+h)(6-x) - x(6-x-h)}{h(6-x-h)(6-x)}}$$ 1

$$= \lim_{h \to 0} \frac{6h}{h(6-x-h)(6-x)}$$

$$= \boxed{\lim_{h \to 0} \frac{6}{(6-x-h)(6-x)}}$$ 1

$$= \boxed{\frac{6}{(6-x)^2}}$$ 1

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 3$.

$$f'(3) = \frac{6}{(6-3)^2} = \frac{6}{9} = \frac{2}{3}$$

$$\boxed{y - 1 = \boxed{\tfrac{2}{3}}(x-3)}$$ 1 1

[c] The position (in inches) of an object moving in along a line is given by $s(t) = \frac{t}{6-t}$, where t is the time in seconds.
Find the instantaneous velocity of the object at time $t = 2$. Give the correct units for your answer.

$$s'(2) = \frac{6}{(6-2)^2} = \frac{6}{16} = \boxed{\frac{3}{8}}\ \boxed{\text{INCHES/SECOND}}$$ 1 1

Using complete sentences & proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: ______ / 2 PTS**

A FUNCTION f IS CONTINUOUS AT a IF

$f(a)$ EXISTS, $\lim_{x \to a} f(x)$ EXISTS AND $\lim_{x \to a} f(x) = f(a)$

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Determine if the function below is continuous at $x = -1$. SCORE: ______ / 4 PTS

State your conclusion clearly, and show whether each condition in the definition of "continuous" is true or false.

In addition, if it is not continuous, determine the type of discontinuity and justify using the appropriate definition.

$$f(x) = \begin{cases} x^2 - x^3 - x^6, & \text{if } x < -1 \\ -1, & \text{if } x = -1 \\ 1 + x^5 - x^7, & \text{if } x > -1 \end{cases}$$

$\frac{1}{2}$ $f(-1) = -1$ EXISTS

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (1 + x^5 - x^7) = 1 - 1 + 1 = 1$ $\frac{1}{2}$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x^2 - x^3 - x^6) = 1 + 1 - 1 = 1$ $\frac{1}{2}$

$\lim_{x \to -1} f(x) = 1$ EXISTS $\frac{1}{2}$ BUT $\neq f(-1)$ $\frac{1}{2}$ NOT CONTINUOUS $\frac{1}{2}$

1 REMOVABLE DISCONT. SINCE $\lim_{x \to -1} f(x)$ EXISTS BUT $\neq f(-1)$

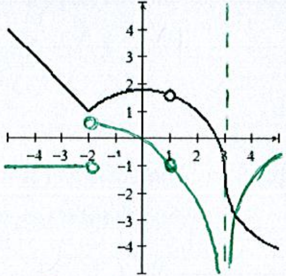
The graph of $f(x)$ is shown on the right. **SCORE: ______ / 6 PTS**

[a] Find all x-coordinates where $f'(x)$ is undefined, and explain **very briefly** why.

1 $\boxed{x = -2 \quad \text{CUSP}}$

1 $\boxed{x = 1 \quad \text{DISCONT.}}$

1 $\boxed{x = 3 \quad \text{VERTICAL T.L.}}$



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[b] Sketch a graph of $f'(x)$ on the same axes.

The time it takes to pay off a certain loan to a friend depends on how much you pay that friend every day. SCORE: ______ / 2 PTS
Suppose $T = f(p)$, where T is the number of days you need to pay your friend, and p is how much you pay every day, in dollars.

[a] What does $f'(5) = -1$ mean ? Your answer must use all the numbers from that equation, and the correct units for those numbers.

IF YOU PAY YOUR FRIEND $5 EACH DAY,
YOUR LOAN WILL BE PAID OFF A DAY SOONER
FOR EACH ADDITIONAL DOLLAR YOU PAY EACH DAY.

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[b] Is there a value p_0 such that $f'(p_0) > 0$? Why or why not ?

NO. PAYING MORE EACH DAY WILL ALWAYS CAUSE YOUR LOAN TO BE PAID OFF SOONER

Using complete sentences & proper mathematical notation, write the formal definition of "derivative (function)". **SCORE: ______ / 1 PT**

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THE DERIVATIVE OF A FUNCTION f IS $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Find the following limits.
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

SCORE: ______ / 7 PTS

[a] $\lim_{x \to \infty} \frac{12}{7 - 3x^{-5}}$

$= \frac{12}{7-3(0)}$

$= \boxed{\frac{12}{7}}$ 1

[b] $\lim_{x \to -\infty} \frac{\sqrt{23x + 2x^6}}{15x^2 - 4x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim_{x \to -\infty} \frac{\sqrt{23x+2x^6}}{15x^2-4x^3} \cdot \boxed{\frac{-\sqrt{\frac{1}{x^6}}}{\frac{1}{x^3}}}$ 1 $\frac{1}{2}$

$= \boxed{\lim_{x \to -\infty} \frac{-\sqrt{\frac{23}{x^5}+2}}{\frac{15}{x}-4}}$ 1

$= -\frac{\sqrt{0+2}}{0-4}$

$= \underbrace{}_{\frac{1}{2}} \boxed{\frac{\sqrt{2}}{4}}$ 1

ANSWER IS POSITIVE

[c] $\lim_{x \to 0^-} \frac{x+3}{e^x - 1}$ $\quad \frac{3}{0^-}$

$= \underbrace{-}_{1} \underbrace{\infty}_{1}$